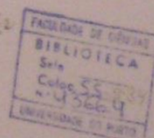


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FUNCTIONAL ANALYSIS AND SEMI-GROUPS

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CHAPTER XII

GENERATION OF SEMI-GROUPS

12.1. Orientation. We come now to the converse problem of the theory: What properties should an operator U possess in order that it be the infinitesimal generator of a continuous semi-group $\mathfrak{S} = \{T(\xi)\}$, $\xi > 0$, of linear bounded operators on a complex (B) -space to itself?

Here the expected type of continuity must be specified. If we require that $T(\xi)$ tend uniformly to I when $\xi \rightarrow 0$, then the solution of the problem is trivial: U must be a linear bounded operator and every such operator generates an analytical group. There are, however, two interpretations of the continuity requirement which lead to worth while problems.

C_1 . $T(\xi)$ tends strongly but not uniformly to I when $\xi \rightarrow 0$.

C_2 . C_1 holds but $T(\xi)$ is uniformly continuous for $\xi > 0$.

We shall obtain sufficient conditions on the spectrum and the resolvent of U in order that the proposed problem have a solution satisfying C_1 or C_2 . The method consists in showing that, under proper conditions, $R(\lambda; U)$ is the Laplace transform of a one-parameter operator $T(\xi)$ having the semi-group property. All three methods used below lead to semi-groups with rather special properties; the conditions imposed on the resolvent are somewhat arbitrary and only in one case can we assert that they are necessary for the desired result. Case C_2 serves as a transition to analytical semi-groups discussed in the next chapter.

There are two paragraphs corresponding to the two problems indicated. There is no literature on this question.

1. GENERATION OF A STRONGLY CONTINUOUS SEMI-GROUP

12.2. $R(\lambda; U)$ as a Laplace transform. In order to obtain an idea of what conditions should be imposed on the operator U , we review the available information concerning the resolvent of the infinitesimal generator of a semi-group $\mathfrak{S} = \{T(\xi)\}$ satisfying condition C_1 .

This condition implies that $T(\xi)$ is strongly continuous for $\xi > 0$; its infinitesimal generator A is a closed unbounded operator whose domain $\mathfrak{D}(A)$ is dense in \mathfrak{X} and the resolvent $R(\lambda; A)$ is the Laplace transform of $T(\xi)$

$$R(\lambda; A)x = \int_0^\infty e^{-\lambda\xi} T(\xi)x d\xi, \quad \lambda = \sigma + i\tau.$$

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